Adaptive Cruise Control: Towards higher traffic flows, at the cost of increased susceptibility to congestion

Kshitij Jerath and Sean N. Brennan*
The Pennsylvania State University

318, Leonhard Building, University Park
PA 16802, USA
Phone: +1-814-863-2430
Fax: +1-814-865-9693
Email: kjerath@psu.edu, sbrennan@psu.edu

Self-organizing traffic jams are known to occur in medium-to-high density traffic flows and it is suspected that ACC algorithms may affect their onset in mixed human-ACC traffic flows. Unfortunately, closed-form solutions that predict the statistical occurrence of these jams in mixed traffic do not exist. In this paper, a closed form solution that explains the impact of adaptive cruise control (ACC) on congestion due to the formation of self-organizing traffic jams (or "phantom" jams) is obtained. The master equation approach is selected for developing a model that describes the self-organizing behavior of traffic flow at a mesoscopic scale. The master equation approach is further developed to incorporate driver (or agent) behavior using ACC or car-following algorithms. The behavior for both human-driven and ACC vehicles is modeled using the General Motors' fourth model. It is found that while introduction of ACC vehicles into traffic may enable higher traffic flows, it also results in disproportionately higher susceptibility of the traffic flow to congestion.

Topics/ Intelligent Transportation Systems, Adaptive Cruise Control & Collision Avoidance, Driver Behavior and Driver Model

* Corresponding author

1. INTRODUCTION

The US Department of Transportation stated in a recent report that "between 1985 and 2006, vehicle miles traveled increased by nearly 100 percent, while highway lane miles only increased 5 percent during the same period" [1]. Another report from the Texas Transportation Institute mentioned that "between 1982 and 2005, the percentage of the major road system that is congested grew from 29 percent to 48 percent" [2]. Recent studies have shown that traffic jams on highways may be *self-organized*, i.e. vehicle clusters may spontaneously emerge from initially homogeneous traffic if the density exceeds a critical value [3]. Such spontaneously-formed vehicle clusters or traffic jams have no apparent root causes (such as an accident) and are often referred to as "phantom" jams [4]. Self-organized traffic jams may lead to adverse effects on the environment (in terms of excessive emissions), financial losses (in terms of fuel wastage) and losses in productivity (in terms of lost man hours). This paper addresses the question of whether Adaptive Cruise Control (ACC) may offer a solution to reduce congestion by preventing self-organized traffic jams.

2. LITERATURE REVIEW

Active research has been performed in the area of adaptive cruise control and car-following driver models by Herman, Gazis and Potts [5], Seiler and Hedrick [6], Darbha [7], Zhou and Peng [8], and Ioannou [9]. Many studies on the impact of automated vehicle systems on traffic flow have been performed in recent times [10]. Studies of Advanced Highway Systems (AHS) with infrastructural support, such as communication networks or dedicated lanes, indicate remarkable improvements in highway capacity [11]. However, since implementing AHS would require an extensive infrastructure overhaul and equally large investments, it is a more realistic goal to expect that highways in the near future will be populated with a mix of ACC and human-driven vehicles. In fact, many major auto manufacturers such as Audi, Ford, Lexus etc. already offer different forms of ACC on their vehicles. Different studies based on systems of mixed ACC and human-driven vehicular traffic suggest that traffic flow may either increase or decrease [10]. Since there isn't a clear mandate on the impact of introduction of ACC vehicles into highway traffic, an urgent need exists to analyze their effect.

Active research has also been performed to analyze the phenomenon of self-organized traffic jams by Kerner and Konhäuser [3], Nagel and Paczuski [12], and Mahnke, Pieret and Kaupužs [13], [14]. Most existing methodologies for analyzing traffic flow are based primarily on either macroscopic or microscopic models. Macroscopic models are not conducive for analyzing traffic comprising a mixture of ACC and human-driven vehicles, while microscopic models rely primarily on numerical simulations and cannot be solved analytically for a large number of vehicles. Further, self-organized traffic jams form at a scale that is between the macroscopic (traffic stream) and microscopic (individual vehicle) scales, and thus a mesoscopic ('meso-', Greek for middle) approach is required to analyze their behavior. Recent advances by Mahnke [13] in modeling the mesoscopic behavior of traffic provide new opportunities for analysis. However, Mahnke primarily focuses on clustering behavior in physical systems [15], and thus little research has been done to analytically study the impact of ACC on the formation of self-organized traffic jams at a mesoscopic scale. Further, other studies regarding the impact of ACC on traffic flow have primarily relied on numerical simulations [16] or experimental studies alone [10]. The following research proposes an analytical framework to overcome the shortcomings of experimental studies and numerical simulation approaches.

3. MASTER EQUATION APPROACH

To simplify the study of highway traffic, the system is often idealized as a closed road of length L with N vehicles on it (Figure 1). Mahnke and Pieret model the formation of clusters, or self-organized traffic jams, in this system as a stochastic process using the discrete form of the master equation [13], and express the growth dynamics of the expected cluster size $\langle n \rangle$ as follows:

$$\frac{d}{dt}\langle n \rangle = \langle w_+(n) \rangle - \langle w_-(n) \rangle \approx w_+\langle n \rangle - w_-\langle n \rangle \quad (1)$$

where $w_+(n)$ denotes the transition probability rate of a vehicle joining a cluster of size n and is dependent on the headway in free flow; $w_-(n)$ denotes the transition probability rate of a vehicle leaving a cluster of size n and is reasonably assumed to be constant $(= 1/\tau)$. The condition for steady state is $w_+\langle n \rangle = w_-\langle n \rangle$. The mean field approximation is used to approximate the expected value of the transition probability rates.

Expressions for free headway obtained from the steady state condition (from equation (1)) and physical constraints (fixed length of track) are equated as follows:

$$\left[h_{free}\right]_{ss} = \frac{L - Nl - (\langle n \rangle - 1)h_{cluster}}{N - \langle n \rangle - 1} \quad (2)$$

where $\left[h_{free}\right]_{ss}$ denotes free headway corresponding to steady state condition and is dependent on the transition probability rates; $h_{cluster}$ denotes the headway inside a cluster and is known from experimental data to be approximately constant [13][14]; and l denotes the length of a vehicle. Additionally, assuming that the cluster size is large, i.e. $n - 1 \approx n$, and rearranging the terms, the following simplified form of the relationship between expected cluster size and traffic density is obtained:

$$\langle n \rangle^* = \frac{\rho^*\left(\left[h_{free}\right]_{ss} + l\right) - l}{\left(\left[h_{free}\right]_{ss} - h_{cluster}\right)} \quad (3)$$

where, $\langle n \rangle^* (= \langle n \rangle l/L)$ denotes the normalized expected cluster size and $\rho^* (= Nl/L)$ denotes the dimensionless density.

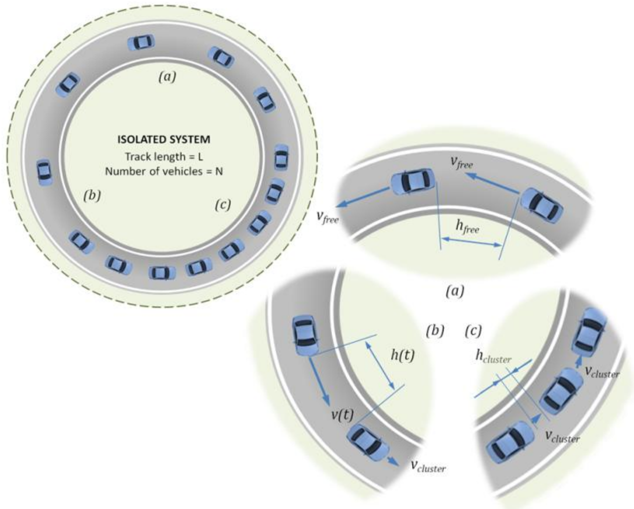

Fig. 1 Closed road system under consideration. (a) Vehicles in free flow; (b) Vehicles transitioning from free flow to jammed state (joining a cluster); (c) Vehicles stuck inside a cluster.

As previously mentioned, the expression for steady state free headway $\left[h_{free}\right]_{ss}$ is obtained using the transition probability rates $w_+(n)$ and $w_-(n)$. Mahnke and Pieret develop an expression for $w_+(n)$ by assuming that vehicles join the cluster by moving at constant speed and "colliding" with the cluster, irrespective of the driver's efforts to maintain a safe velocity and distance from the preceding vehicle during the "collision" process. This does not reflect the true driver behavior while approaching a cluster. Instead, in this study, new transition rates are determined based on car-following or ACC algorithms to more accurately describe driver behavior.

4. NEW TRANSITION PROBABILITY RATES

In the present study, new transition rates are derived based on car-following models to represent driver behavior. One of the most popular and validated, yet simple, car-following models is the fourth model proposed by the General Motors research group [5], [17]. The model is used in the present analysis and is described as follows:

$$\ddot{x}_{n+1}(t+\Delta t) = \frac{\alpha\,[\dot{x}_{n+1}(t+\Delta t)]}{[x_n(t) - x_{n+1}(t)]}\left(\dot{x}_n(t) - \dot{x}_{n+1}(t)\right) \quad (4)$$

where $x_{n+1}(t)$ denotes the position of the vehicle entering the cluster, $x_n(t)$ denotes the position of the tailing vehicle in the cluster, and α denotes the sensitivity of the driver of the vehicle entering the cluster. The headway $x_n(t) - x_{n+1}(t)$ is represented by $h(t)$. The range of driver sensitivities is determined using typical traffic conditions and comfortable deceleration standards set by AASHTO. The typical traffic flow is assumed to have free flow velocity of about 25 m/s (about 55 miles/hour), free headway of about 100 m, and cluster velocity of about 0-2 m/s. Further, the maximum permissible deceleration is limited to 3.4 m/s^2, according to AASHTO standards. Using these values, the process of a vehicle joining a cluster is simulated to obtain the maximum observed deceleration. The simulation outputs are included in figure 2(a) for two different driver sensitivities. The simulation is repeated for various values of driver sensitivities and the acceptable driver sensitivities are determined to fall in the approximate range [0.4, 0.65] (Figure 2(b)).

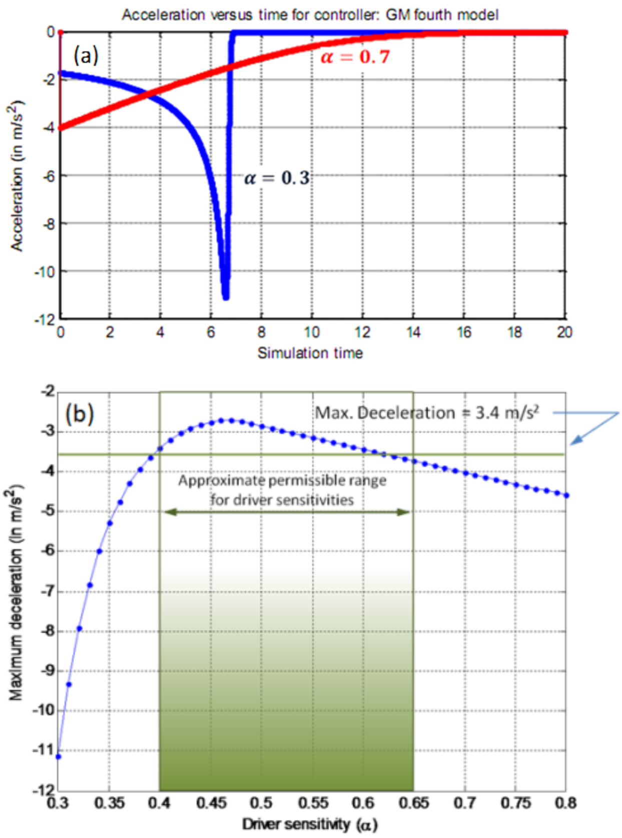

Fig. 2 (a) Maximum observed deceleration during simulation of a vehicle joining a cluster with typical traffic conditions. (b) Maximum observed deceleration with varying driver sensitivities.

Next, the equation of the car-following model is solved to determine the time taken to join the cluster (t_{join}). The resulting expression for t_{join} is a hypergeometric series with no closed-form solutions:

$$t_{join} = \frac{1}{v_c}\sum_{m=1}^{\infty}\left\{\frac{1}{1-m\alpha}\left(\frac{v_c}{k}\right)^m \left(h_{free}^{1-m\alpha} - h_{cluster}^{1-m\alpha}\right)\right\} \quad (5)$$

where k is a driver dependent constant and v_c is the velocity of the tailing vehicle in the cluster. However, it is observed that as an increasing number of terms is included in calculating t_{join}, the hypergeometric series converges quickly to the true solution obtained from numerical simulation (Figure 3(a)). An additional key insight of this paper is to recognize that the hypergeometric series is constrained by the range of permissible driver sensitivities. Considering the range of permissible driver sensitivities, it is observed that one can approximate a closed-form solution by using the first term of the series (t_1) and an appropriate truncation ratio (T_R), such that $t_{join} \approx T_R t_1$. For typical traffic conditions, the truncation ratio (T_R) is determined to be about 1.4 (Figure 3(b)).

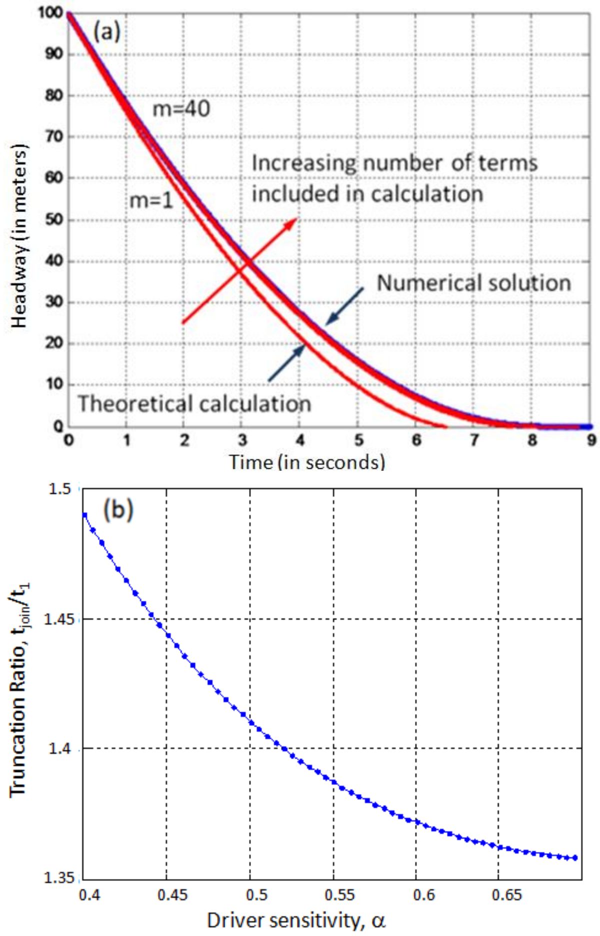

Fig. 3 (a) Time to join a cluster – theoretical expression converges quickly to the numerical solution (b) Range of permissible driver sensitivities limits the variation of truncation ratio, $T_R = t_{join}/t_1$.

The transition probability rate for joining the cluster is thus given by the inverse of time taken to join a cluster, as shown in equation (6). The growth dynamics for the cluster are determined as proposed in [13], but the closed-form approximations of the new transition rates are used to account for the car-following behavior.

$$w_+(n) = \frac{1}{\bar{T}_R\, t_1} = \frac{k(1-\alpha)}{\bar{T}_R}\left(\frac{1}{h_{free}^{1-\alpha} - h_{cluster}^{1-\alpha}}\right) \qquad (6)$$

The expression for steady state free headway is thus obtained from equations (1) and (6), and is included as equation (7). The expression may be substituted in equation (3) to obtain the relationship between normalized expected cluster size and dimensionless density. Thus, the new transition probability rates yield a new expression for steady state free headway which better reflects the driver behavior and consequently the true cluster formation process.

$$[h_{free}]_{ss} = \left\{h_{clust}^{1-\alpha} + \frac{\tau k(1-\alpha)}{\bar{T}_R}\right\}^{1/1-\alpha} \qquad (7)$$

5. INTRODUCTION OF ACC VEHICLES INTO TRAFFIC FLOW

The closed-ring system is now considered with traffic containing a mixture of human-driven vehicles and ACC vehicles. Let the proportion of ACC vehicles on the closed road be p. If the population of vehicles is large, such that the proportion of human-driven and ACC vehicles is approximately the same both inside and outside the cluster, then the effective transition probability rates are given by:

$$w_+^{eff}(n) = (1-p)w_+^{H}(n) + p\,w_+^{ACC}(n); \qquad w_-^{eff}(n) = 1/\tau \qquad (8)$$

where $w_+^H(n)$ denotes transition probability rate of joining a cluster for a human-driven vehicle with $\alpha = 0.4$, and $w_+^{ACC}(n)$ denotes transition probability rate of joining a cluster for an ACC vehicle with $\alpha = 0.7$. The sensitivity value for human drivers is supported by experimental data from German highways [14]. If $p = 0$, i.e. the traffic consists only of human-driven vehicles, each with driver sensitivity $\alpha = 0.4$, then the dimensionless critical density obtained from the above theoretical analysis is approximately 0.1 (Figure 4(a)). This value is in reasonable agreement with the actual value of critical density observed on German highways with human-driven vehicles (Figure 4(b)).

The sensitivity value for ACC vehicles is determined from the necessity of obtaining a closed form solution for the analysis. When the expressions for individual transitional probabilities $w_+^H(n)$ and $w_+^{ACC}(n)$ are substituted in the effective transition probability rate $w_+^{eff}(n)$ and the steady state condition $w_+^{eff}\langle n\rangle = w_-^{eff}\langle n\rangle$ is considered, the following equation is obtained:

$$h_{free}^{1-\alpha_H} h_{free}^{1-\alpha_{ACC}} - b\,h_{free}^{1-\alpha_H} - c\,h_{free}^{1-\alpha_{ACC}} + d = 0 \qquad (9)$$

where b, c, and d are functions of $h_{cluster}$, τ, p, α_{ACC}, α_H, and T_R. In the depicted general form, equation (9) is a transcendental equation and can be solved using only numerical or graphical methods. In order to obtain an analytical solution for the steady state free headway, the transcendental equation is reduced to an algebraic equation (quadratic, cubic or bi-quadratic) by enforcing a constraining relation on the values that α_H and α_{ACC} may simultaneously assume. One such relation that reduces equation (9) into a cubic equation and thus allows a closed form solution is $(1-\alpha_H) = 2(1-\alpha_{ACC})$, $i.e.\, \alpha_{ACC} = 0.5(1+\alpha_H)$. It may be observed that, once this substitution is made, arbitrary choices of driver sensitivities cannot be made in this analysis. This is due to the fact that the choices are restricted by two constraints, viz. the maximum acceptable deceleration (as discussed in section 4.2), and the need to obtain a closed form solution. A number of values of driver sensitivities (α_H, α_{ACC}) such as (0.35, 0.675), (0.4, 0.7) etc. which satisfy the relation $(1-\alpha_H) = 2(1-\alpha_{ACC})$ also lie approximately in the range defined by maximum acceptable deceleration based on AASHTO standards.

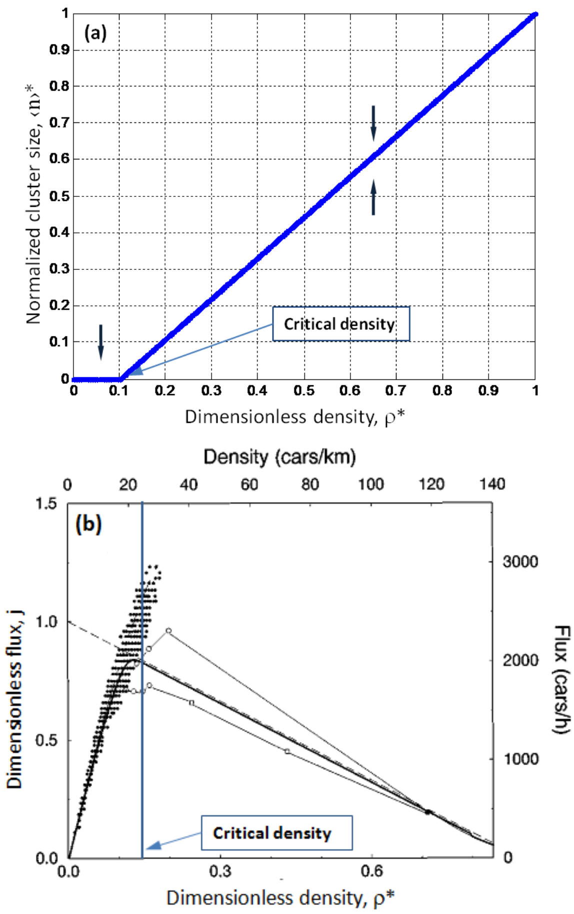

Fig. 4 (a) Phase portrait for cluster size versus density for traffic consisting of human-driven vehicles with $\alpha = 0.4$. (b) Experimental data for traffic flow consisting solely of human drivers [Modified from [14], data from [3]]

Thus, the relation $(1-\alpha_H) = 2(1-\alpha_{ACC})$ may be used as an approximation, together with this restricted set of values, to reduce equation (9) into a cubic form.

The expression for steady state free headway in a multi-species environment is obtained by solving the cubic equation and then substituted into equation (3) to obtain a relationship between expected cluster size and density in a multi-species environment. The results from the analysis are discussed next.

6. RESULTS AND SIMULATIONS

The expression for steady state free headway in a multi-species environment is substituted in equation (3) to obtain the relationship between expected cluster size and density. The phase portraits are obtained for the analytical solutions for both human and ACC driver models. Figure 5(a) indicates that, if the vehicle population consists of only ACC drivers $(p = 1, \alpha = 0.7)$, the traffic operates at *higher* critical densities, and consequently *higher* traffic flows, as compared to when it consists of only human drivers $(p = 0, \alpha = 0.4)$. The relationship between cluster size and density in a single-species environment is validated using a Monte Carlo simulation wherein the cluster formation process is modeled as a one-dimensional random walk (Figure 5(b)). The simulation is valid only up to dimensionless density $\rho^* = 0.8$ in this case, beyond which the expression for free headway obtained from physical constraints (fixed length of road) yields incorrect results, due to finite length of vehicles (l).

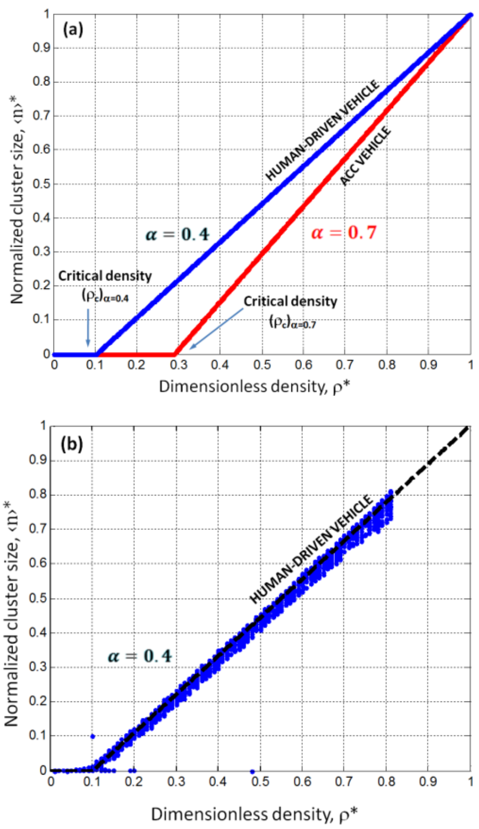

Fig. 5 Phase portraits for cluster size versus density. (a) Analytical solution for human and ACC drivers; (b) Monte Carlo simulation and comparison with analytical solution for human drivers.
Lines indicate analytical solution. Solid dots indicate simulation results.

Further, it is found that as the proportion of ACC vehicles on the road is increased, the traffic flow becomes increasingly sensitive to changes in vehicle population proportions. In predominantly human driver traffic in the jam-free regime, a small change in vehicle proportion does not change the state of the traffic flow, which continues to operate in the jam-free regime (Figure 6(a) - operating point A_1). On the other hand, if the same change of vehicle proportion is introduced in predominantly ACC traffic, it causes the traffic flow to change from a jam-free state to a self-organized jam or congested state (Figure 3(a) - operating point B_1). Figure 6(b) describes the sensitivity of critical density to proportion of ACC vehicles and indicates that traffic systems with very high ACC penetration are up to 10 times as susceptible to congestion caused by self-organized traffic jams, as traffic systems with very low ACC penetration.

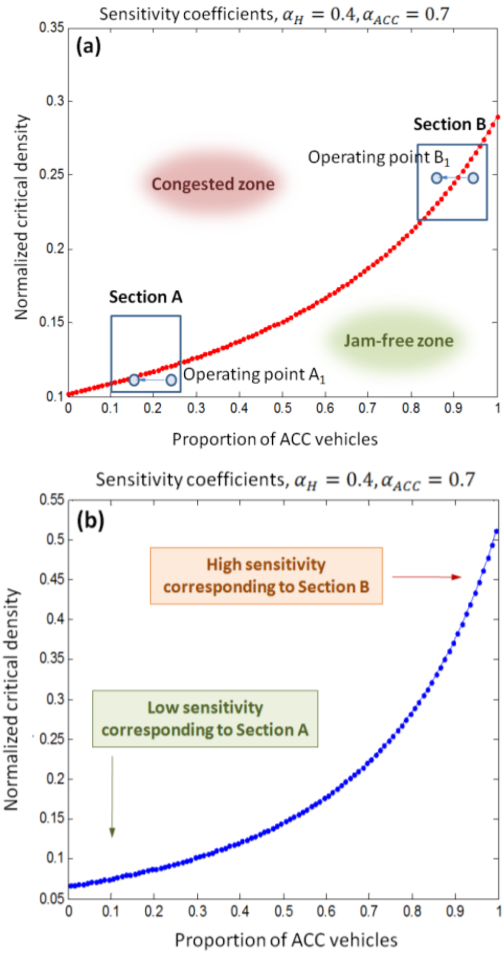

Fig. 6 Increased ACC penetration results in an increase in critical density at which self-organized jams begin to form. (a) Critical density vs. ACC penetration. (b) Sensitivity of critical density to proportion of ACC vehicles on the road.

Monte Carlo simulation of the multi-species system is used to determine the normalized critical density as the proportion of ACC vehicle on the road increases. Specifically, the Monte Carlo simulation is used to determine the lower bound of the density, or the density at which clusters begin to form. The results from the Monte Carlo simulation appear to match quite well with the theoretical analysis, as shown in Figure 7.

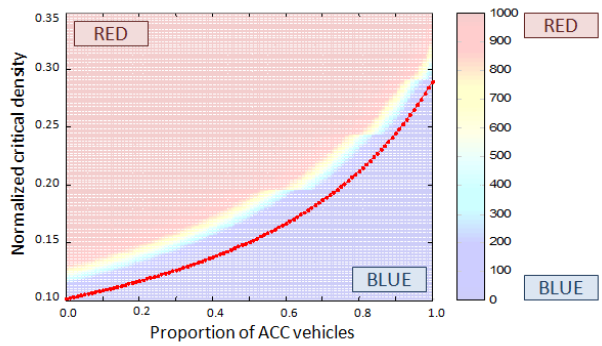

Fig. 7 Results from the Monte Carlo simulation appear to agree with the theoretical analysis. Colormap indicates the total number of vehicles inside a cluster.

7. CONCLUSIONS

The study has shown that as the percentage of ACC vehicles in the traffic system is increased the critical density also increases correspondingly. The increase in critical density implies that the density at which vehicle clusters begin to spontaneously appear is increased. This indicates that the traffic flow can operate at higher densities and consequently higher flow rates, since it is known from the fundamental diagram of traffic flow that, in the free flow regime, the flow increases as the density increases.

Additionally, while increased ACC penetration may allow the traffic system to operate at increased densities and flows, it comes at a cost. As ACC penetration increases, a small percentage of drivers with low sensitivities are enough to cause a self-organized traffic jam. In other words, in a predominantly ACC traffic system, introduction of a small percentage of human drivers may cause a rapid reduction of critical density, resulting in a self-organized traffic jam.

REFERENCES

[1] Federal Highway Administration (FHWA), *Our nation's highways 2008.* Washington D.C. : US DoT, 2008.
[2] D.L. Schrank and T.J. Tomax, "The 2007 Urban Mobility Report", Texas Transportation Institute, 2007.
[3] B.S. Kerner and P Konhäuser, "Cluster effect in initially homogeneous traffic flow", *Phys. Rev. E*, vol. 48, pp. 2335-2338, 1993.
[4] Y. Sugiyama et al., "Traffic jams without bottlenecks - experimental evidence for the physical mechanism of the formation of a jam" *New Journal of Physics*, vol. 10., 2008.
[5] D.C. Gazis, R. Herman and R.B. Potts, "Car-Following Theory of Steady-State Traffic Flow", *Operations Research*, vol. 7, pp. 499-505, 1959.
[6] P. Seiler, B. Song and J. K. Hedrick, "Development of a collision avoidance system", *SAE SPEC. PUBL.*, vol. 1332, pp. 97-103, 1998.
[7] S. Darbha and K. R. Rajagopal, "Intelligent Cruise Control Systems and Traffic Flow Stability", California Partners for Advanced Transit and Highways, 1998.
[8] J. Zhou and H. Peng, "Range Policy for Adaptive Cruise Control Vehicles for Improved Flow Stability and String Stability", *IEEE Transactions on Intelligent Transportation Systems*, vol. 6, pp. 229-237, 2004.
[9] P. A. Ioannou and C. C. Chien, "Autonomous intelligent cruise control", 1993, *IEEE Transactions on Vehicular Technology* ,vol. 42, pp. 657-672, 1993.
[10] P. J. Zwaneveld and B. van Arem, "Traffic effects of automated vehicle guidance systems", Department of Traffic and Transport, Netherlands Organization for Applied Scientific Research, 1997.
[11] S. E. Shladover, "Potential freeway capacity effects of Advanced Vehicle Control Systems", *Proc. Of 2nd International Conference on Applications of Advanced Technologies in Transportation Engineering*, Minneapolis, Minnesota, USA, 1991.
[12] K. Nagel and M. Paczuski, "Emergent traffic jams", *Physical Review E*,vol. 51, pp. 2909-2918, 1995.
[13] R. Mahnke and N. Pieret, "Stochastic master-equation approach to aggregation in freeway traffic", *Phys. Rev. E*,vol. 56, pp. 2666-2671, 1997.
[14] R. Mahnke and J. Kaupužs, "Stochastic theory of freeway traffic", *Phys. Rev. E*, vol. 59, pp. 117-125, 1999.
[15] J. Schmelzer, G Röpke, R. Mahnke, *Aggregation Phenomenon in Complex Systems*, Wiley-VCH, 1999.
[16] A. Kesting et al." Jam-avoiding adaptive cruise control (ACC) and its impact on traffic dynamics" [ed.] A Schadschneider, et al. *Traffic and Granular Flow*, pp. 633-643, Berlin : Springer, 2005.
[17] A. D. May, *Traffic Flow Fundamentals*, Prentice-Hall, Inc., 1990.